



Mathmatics in the Biosciences

Maths is essential in bioscience research, and cross-curricular links between biology and maths benefit the learning of both subjects. Mathematical skills are increasingly required by biologists tackling complex problems and analysing large amounts of data. This resource will provide contexts for learning essential maths techniques and will enrich science lessons by outlining the way maths is used in cutting-edge biological research.







Key Information

Teacher

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Key Information

View online



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Science Topics

Sampling, environmental observations, disease, animal behaviour, calculating speed, populations, experimental design, choice tests, crop pests

Maths Topics

Vectors, symmetry, geometry, differential equations, division, Pythagoras' theorem, coordinates, factorials, statistics, probability.

Resources

Age	Recent research
11-18 years old	Maths problems
	Online activities
	• Guide to differentiation
	Glossary

- Curriculum guide
- Classroom PowerPoint slides





Key Information

Keywords

Modelling, sampling, environment, observation, disease, behaviour, speed, population, pest, vector, symmetry, geometry, equation, division, Pythagoras, coordinate, factorial, statistics, flight, flower, pollination, foraging, travelling salesman, radar, tracking, permutations, experimental design, slug, replication, randomisation, blocking, neighbour balance, ash dieback, fungus, epidemic, mark-recapture, bees, habitats, pesticides, conservation, estimation, distribution, variance, variability, standard deviation.



Introduction

BBSRC is committed to raising the mathematical and computational skills of biologists at all levels.

The application of mathematics to biological problems and large data sets is essential for future progress in the biosciences. Mathematical biology has a range of applications in biology, medicine and biotechnology through modelling, systems biology, bioinformatics and computational biology. Advances in computing, networking, next-generation sequencing and imaging allow scientists to carry out cutting-edge research and this is revolutionising the biosciences. For instance, the development of an algorithm, based on the mathematics used in ecological mark-recapture estimates of population size, combined with full genome sequencing technologies, could enable doctors to sequence and analyse the DNA in your genome while you wait. Using new technologies, BBSRC is enabling these innovative new ways of working.





Recent Research

Teacher

Strategic sampling - find the infected ash tree with vectors

At Rothamsted Research, scientists are using mathematics to predict and prevent tree diseases including ash dieback and sudden oak death.

Trees in the UK face an uncertain future with climate change bringing more and more extreme weather, and a recent rise in exotic pests and diseases coming into the country.

The number of new diseases entering the UK has risen dramatically in the past decade.

In February 2012 a highly infectious disease known as ash dieback was found in UK woodlands. This devastating disease, caused by a fungus called Chalara fraxinea, threatens ash trees and forests across the country, with ash being the third most common broadleaf tree in the UK (after oak and birch).

Ash dieback, which first appeared in Poland in the early 1990s, has spread throughout continental Europe over the past 20 years, and in Denmark ash dieback has wiped out 60 to 90 per cent of ash woodland. Infection leads to leaf loss and usually death of the tree,



Breeding resistant ash trees

© Prof. Erik Dahl Kjær. Forest & Landscape, Faculty of Life Sciences, University of Copenhagen.

causing extensive economic and environmental damage. Little is known about the fungus, and why it is so aggressive, or its interactions with the trees that it attacks. This prevents effective control strategies.

Scientists at Rothamsted Research are using mathematical models to work out how ash dieback will spread across the UK and to investigate questions such as: what is the probability of finding an invading epidemic before it becomes too widespread to stop?

Computer models will help plan monitoring of the distribution and spread of the fungus, as well as charting how the disease might progress. This knowledge will help to fight the fungus and to replace lost trees with those more able to survive.

Effective surveillance is crucial in order to discover new diseases before they are out of control. However, finding the small handful of diseased trees that start an epidemic in the vast UK landscape can be like trying to find a needle in a haystack. Predictive models of epidemics can identify which areas in the UK are most at risk, helping to prioritise areas for surveillance and visits to look for diseases.

Early detection of new disease outbreaks is crucial in combating epidemics and discovering solutions to infections. Volunteer groups such as the Tree Wardens, who have a good understanding of trees and woodland environments and the issues that plague them, have a strong potential role to play in finding and reporting new tree disease outbreaks.

There are other diseases of trees spreading in the UK such as Phytophthora ramorum, a pathogen known as sudden oak death, which is a significant problem for commercial larch trees, and acute oak decline, a disease that has been linked to a bacterial species that can kill oak trees.





Recent Research

Sudden oak death kills oak and other species of tree and has had devastating effects on the tree populations in California, Oregon and more recently in Europe, including the UK. Symptoms include bleeding cankers on the tree's trunk and dieback of the foliage, in many cases eventually leading to the death of the tree.

Phytophthora ramorum also infects a great number of other plant species, significantly rhododendrons, causing a non-fatal foliage disease known as ramorum dieback. Such plants can act as a source of the inoculum of the disease, with the pathogen producing spores that can be transmitted by wind and rainwater.

Overseas transport of ornamental plants often results in this disease spreading to plant nurseries and from there into the natural wildlife. Once the disease zis detected within a nursery, steps are taken to ensure that all the potential carriers of the disease are eradicated. However, sometimes the disease has already spread to the local wildlife before this can be done.

At Rothamsted Research, scientists are trying to develop mathematical models which they can use to provide government with a sampling strategy to check whether the disease has indeed spread to outside the nursery. Because sampling for disease is costly and very time-consuming, the scientists need to come up with very efficient sampling strategies.



Bioscience to help battle ash dieback www.rothamsted.ac.uk/news/bioscience-help-battle-ash-dieback





Science topics

Sampling, environmental observations, disease, ecosystems

Maths topics

Coordinates, vectors, Pythagoras' theorem

Learning outcomes

Age	Students will be able to:
14-16 years old	 Use coordinates to describe the position of a point on a grid Represent column vectors Calculate the length of a hypotenuse
Duration	
45 minutes	

Keywords

Column vector, coordinate, ash dieback, disease, fungus, epidemic

Prior learning

What you will need

Students should be able to provide a position on a grid as a coordinate and carry out addition of the pair of numbers in a coordinate.

- Two game boards per group
- Scientist counters
- Disease tree
 counters
- Strategy sheets
- Pens







Introducing the lesson

This lesson is designed to be delivered as either a maths or science lesson. The strategic sampling game is related to the work done at Rothamsted Research on tree diseases.

Provide the students with the recent research information along with the activity sheets. Students can check their results using the answer sheets provided at the end.

In this interactive session the students are going to try to develop a strategy for finding diseased trees within an ash plantation. As with all the other activities, the aim is to make the activity as interactive as possible. So rather than giving the students answers straight away, ask some questions and coach the students in the right direction. The activity is best carried out like the popular game Battleships, with each team ensuring that their board cannot be seen by their opponents.

Explain to the class what a column vector is and how to write one down.

Game play

- Divide the class into groups of two to four pupils
- Divide each group into two teams: team 'scientist' and team 'ash dieback'
- Provide each team with a game board and counters
- Step 1: team 'ash dieback' hide the infected tree in the plantation by marking it on their sheet. Make sure that they are aware that developing a strategy is very important. If they know what the strategy is this might help them in deciding where not to hide the infected tree. Note that each intersection point on the grid represents trees that can contain the ash dieback infection
- Step 2: team 'scientist' make their first guess and place a scientist on the relevant grid point. Make sure you query them about their choice to check whether they are actually working out a strategy
- Step 3: team 'ash dieback' work out the minimum number of horizontal and vertical steps required to reach the infected tree from the location of the first guess. The team then reveal the total number of steps
- Step 4: repeat steps 2 and 3 until the infected tree has been located
- Step 5: team 'scientist' and team 'ash dieback' swap roles and repeat the game
- Step 6: have the students write down the minimum number of steps required to find the infected tree and an explanation of their strategy for finding it

The students should have a go at playing both the scientist and the infected tree. If after this they have not worked out a strategy, challenge them yourself.

With the correct strategy, the infected tree can always be found within three guesses. If things seem too easy, extend the game by introducing an area on the grid that they can only enter for the final guess, i.e. when they know where the infected tree is hiding. This could represent the scientist normally only walking across the main path through the plantation.





Plenary

The 'find the ash dieback' game is a simplification of the real approach scientists use to carry out strategic sampling. In real life a scientist does not get information on how far away the disease is from his/her previous sampling point. Instead the scientists at Rothamsted rely solely on their knowledge of the disease and its spread and the distribution of the plants that can be infected by the disease (and nifty maths and statistics techniques, of course). Even though this game is not directly applicable to disease management it will help the students think about developing strategies, which in all disease management work is the initial starting point. If you haven't worked out a good strategy, you have no idea where to start with the maths.

Video



Fraxinus: a Facebook game to crowdsource the fight against ash dieback. www.youtube.com/watch?feature=player_embedded&v=JxqXeijQOOM





Curriculum Links

DRAFT Science programme of study for Key Stage 4

Health, disease and the development of medicines

- The relationship between health and disease
- Different types of disease infectious and non-communicable diseases (NCDs) and the interactions between them
- The role of bacteria, viruses and fungi in causing infectious disease in animals and plants

Ecosystems

- Levels of organisation: species, population, community, ecosystem, biome and biosphere
- Components of an ecosystem (abiotic factors and biotic community)
- Relationships among organisms in an ecosystem

Human interactions with ecosystems

- The importance of biodiversity in ecosystems
- Identifying and classifying local species and using keys
- Measuring the distribution, frequency and abundance of species in a range of habitats and explaining outcomes in terms of abiotic and biotic factors
- Measuring changes in the distribution and abundance of organisms as a way of measuring and monitoring change in ecosystems
- Positive and negative human interactions with ecosystems

DRAFT Maths programme of study for Key Stage 4

Geometry and measures

• Apply Pythagoras' theorem and trigonometric ratios to find angles and lengths in right-angled triangles







Further reading and links

Using mathematics to predict and prevent tree diseases [Reference/webpage no longer available – January 2017] Bioscience to help battle ash dieback [Reference/webpage no longer available – January 2017] Genome sequence for ash dieback survivor [Reference/webpage no longer available – January 2017] Gamers to join ash dieback fight-back [Reference/webpage no longer available – January 2017]

www.forestry.gov.uk/chalara www.ashtag.org

Research groups

Professor Christopher Gilligan, Cambridge University www.plantsci.cam.ac.uk/research/chrisgilligan Dr Nik Cunniffe, Cambridge University www.plantsci.cam.ac.uk/research/nikcunniffe





Game Board and Counters

Student





Instructions:

Cut out the game board, infected tree and scientist game counters



www.bbsrc.ac.uk



Diseases reduce the yield of crops such as food and timber, and it is therefore important to quickly detect diseased plants. Searching for disease costs time and money so it has to be done efficiently. At Rothamsted Research, scientists develop strategies for detecting new diseases quickly and efficiently.

Find the ash dieback game

A disease has entered an ash plantation. The trees are grown in ordered lines to make it easier to harvest them but which tree is infected by the disease?

To test your skills in strategic sampling try the ash dieback game to see how easily you can find the infected tree.

The trees are equally spaced 1 metre apart along a grid. Can you develop an optimal search strategy? A minimum of two players is required for this game. Players are either part of team **'ash dieback'** or team **'scientist'**.

Instructions

Team 'ash dieback'

- 1. Place the infected tree on the game board at one of the intersection points of the grid. DO NOT LET THE OTHER TEAM SEE!
- 2. After each guess by team '**scientist**', tell them the minimum number of steps along the grid they will need to take to find the infected tree the scientist can only move **horizontally** and **vertically**
- 3. Add up the number of horizontal and vertical steps along the grid that the scientists would need to move, and write this as x + y = n. Tell team 'scientist' the value of n

Are some places better than others for hiding from the scientist?

Team 'scientist'

- 1. Select an intersection on the grid where you think the infected tree might be hiding
- 2. Place a scientist on this intersection and give the position to team 'ash dieback'
- 3. After each guess, team **'ash dieback'** will give you the shortest distance you'd have to walk to find the infected tree your scientist can only move **horizontally** and **vertically**
- 4. Write down the column vector for where your next guess for the location of the infected tree will be

Can you come up with a reliable strategy for choosing intersections that will locate the disease in the minimum number of guesses?

Question 1: The disease can always be found in a minimum of how many guesses? **Question 2:** How would you explain your strategy to other students?





Student Sheets

Hints

Team 'ash dieback'

Try to anticipate what the scientist will do in an effort to find the diseased tree. Make sure you avoid areas where team **'scientist'** are likely to make their initial guesses. Ash dieback does not always infect trees in places that are easy to find.

Team 'scientist'

Try to decide which places are good for first guesses.

When team **'ash dieback'** give you the results of your first guess, identify all the possible places where the disease could be. What pattern does this make? Use pen and paper to try and work it out.

Have another guess and repeat to find all the possible places where the disease could be.

Now use the information from both guesses to find some overlapping points.

Continue in the same way ...

Game rules

- Divide into two teams: team 'scientist' and team 'ash dieback'.
- Step 1: team 'ash dieback' hide the infected tree in the orchard by marking it on their sheet. Note that each intersection point on the grid where there is a tree can contain the ash dieback infection.
- **Step 2:** team **'scientist'** make their first guess and place a scientist on the relevant grid point. Is your team actually working out a strategy like the Rothamsted scientists?
- Step 3: team 'ash dieback' work out the minimum number of horizontal and vertical steps required to reach the infected tree from the location of the first guess. The team then reveal the total number of steps.
- Step 4: repeat steps 2 and 3 until the infected tree has been located. Each time team 'scientist' make a guess they must record the column vector for their next location to keep track of their movements.

Coordinates and column vectors

Coordinates are pairs of numbers used to describe the position of a point on a grid.

The first number is the distance moved along the x-axis. The second number is the distance moved along the y-axis. The x-axis and y-axis cross at the origin, 0. The coordinates (x,y) describe the position of a point x units across, left or right, and y units up or down from the origin.

Vectors are quantities that have both size and direction. A **column vector** describes both the distance and the direction. When writing a column vector the top number represents the distance moved **horizontally** (+ to the right, - to the left). The bottom number represents the distance moved **vertically** (+ upwards, - downwards).





Question 1: The disease can always be found in a minimum of how many guesses? The infected tree can be found in a minimum of three guesses.

Explanation: As your first point pick the coordinate (1,1). The possible places where the disease could be hiding are the coordinates that add up to the number indicating how far away from the disease you are.

For example, if the distance is 6 the disease could be at the coordinates (1,7), (2,6), (3,5), (4,4), (5,3), (6,2), (7,1).

Notice that all these points lie on a straight line (see grey line in diagram below) and the infected tree must be somewhere along this line. This has reduced the search area from 2D to 1D.

If you now choose a point at one end of the line (column vector 6,0 or 0,6) and receive the next distance value from team **'ash dieback'** it will help you navigate to a specific point on the straight line, where the disease is hiding.

In the example below the next distance is 8 grid squares. Only one tree along the grey line is 8 grid squares away. The column vector to the infected tree is (4,4) and its coordinate is (3,5).

This explanation can be illustrated by the diagrams below.











With this method the disease can always be located in three guesses. By starting in the centre it would always take four guesses to locate the disease.

How would the best strategy change if all guesses (apart from the final guess) would have to be made within a restricted area of the grid? Would this increase the number of guesses required and if so by how many?

When developing this strategy you had some prior knowledge in the sense that after each guess you were told how far away from the disease you were. In reality this is often not the case which drastically complicates the development of optimal strategies. Rothamsted Research tries to solve these types of complicated optimisation problems.

Extension activity

The scientists have been asked to investigate another plantation where unfortunately the disease is now spreading. Two trees have been found to be infected and the scientists want to work out how far the disease has spread from one tree to the next.

The disease can be spread directly to other trees by the wind. However the scientists can only move horizontally and vertically around the plantation, so they need to work out how far it is from an infected tree to the next in a straight line.

You can calculate this using Pythagoras' theorem.

Pythagoras' theorem – in any right-angled triangle the square of the length of the hypotenuse (h) is equal to the sum of the squares on the two shorter sides (a and b).

The coordinates of the first infected tree is (2,1) and the vector to the second infected tree is (8,3)

Remember each grid is 1 metre by 1 metre.

Question 3: What is the direct distance from the first infected tree to the second infected tree in metres? Express the answer to two significant figures.

Answer: 6.7 metres







Glossary 🖬

Ash dieback

Chalara dieback of ash is a serious disease of ash trees caused by a fungus called Chalara fraxinea.

Coordinate

One or two numbers used to define a point in space.

Epidemic

An outbreak of a contagious disease that spreads rapidly and widely.

Fungus

Eukaryotic organisms of the kingdom Fungi, which lack chlorophyll and vascular tissue and range in form from a single cell to a body mass of branched filamentous hyphae that often produce specialized fruiting bodies. The kingdom includes the yeasts, moulds, smuts, and mushrooms.

Mathematical model

A mathematical model is a description of a system using mathematical concepts and language.

Pythagoras' theorem

The theorem applies to any right-angled triangle and states that the square of the length of the hypotenuse (the side opposite the right angle) is equal to the sum of the squares of the other two sides: $h^2 = a^2 + b^2$.

Strategic sampling

A statistical approach to sampling is used in order to ensure a more representative sample. The sample is not drawn at random from the whole population, but separately from a number of smaller groups of the population.

Vector

A quantity, such as velocity, completely specified by a magnitude and a direction.







Recent Research

Teacher

Observing behaviour - measuring bee flight using Pythagoras' theorem

Scientists have discovered that bumblebees are able to find the shortest distance between flowers when collecting nectar. When bumblebees have to visit more than one or two flowers they have to decide on the best route to take. They can remember the locations of flowers so they can choose the shortest route even if they discover the flowers in a different order.

Finding the shortest route between a number of locations is known as the travelling salesman problem. It is a complex mathematical problem which keeps supercomputers busy for days, but bumblebees are able to solve the problem even though they have a brain the size of a grass seed.



Bumblebee with radar tag

© Rothamsted Research

In nature, bees have to visit hundreds of flowers to collect nectar and do it in a way that minimises travel distance before finding their way

home. Foraging bees solve travelling salesman problems every day using a relatively tiny number of brain cells. They visit flowers at multiple locations and, because bees use lots of energy to fly, they find a route which keeps flying to a minimum.

Computers solve this problem by comparing the length of all possible routes and choosing the shortest. By understanding how bees can solve the problem with such a tiny brain we can improve the management of everyday networks, such as the management of traffic on the roads, without needing lots of computer time.

To study how the brain processes complex tasks, a team of biologists, mathematicians and physicists have been carrying out some amazing experiments. The scientists used computer-controlled artificial flowers to test whether bees would follow a route defined by the order in which they discovered the flowers or if they would find the shortest route. After exploring the location of the flowers, bees quickly learnt to fly the shortest route.

In the early experiments the bees had to find their way around four artificial flowers in a large flight room. They were observed visiting each flower and the time spent flying and the distance they travelled was recorded. Following this success, the scientists decided to monitor the bumblebees flying around a small field as they visited five artificial flowers. To do this, the scientists used radar tracking and motion-triggered webcams. Bumblebees were fitted with tiny radar transponders, which were stuck on the bees' backs with double-sided tape.

The experiment was carried out in October when there were few natural sources of nectar and pollen and the bees were more likely to focus on the artificial flowers. The scientists arranged the flowers in a pentagon and spaced them 50 metres apart – a distance more than three times as far as bumblebees can see, so the bees had to actively fly around to locate their next target. A motion-triggered webcam was attached to each flower to record the bees' visits. Then, every day for a month, each bee was freed to forage for 7 hours.





Recent Research

The scientists tracked the sequence of flower visits the bees made each time they went foraging. The bumblebees found the closest flowers first and added new flowers during subsequent trips, remembering which route was the shortest.

Using trial and error the bees learnt to adjust their routes to find shorter paths. The bees did not try every possible route between the flowers, but were able to make efficient trips after trying only about 20 of the 120 possible routes. In contrast to computers, the bees did not find the absolute shortest route in the simple experimental arrangement. But they came very close, especially considering that they explored only a small fraction of the possible routes and did so very quickly.

Brains are nature's computers, and by studying the tiny brains of bees, scientists may be able to identify the minimal neural circuitry required for solving difficult mathematical problems. Improvements in understanding complex problem-solving could lead to the creation of faster computers.

The research also gives an insight into bumblebee behaviour. Bumblebees play a vital role in pollinating certain food crops and so an understanding of how they forage is important for future food security.

There is a common perception that smaller brains constrain animals to being simple reflex machines. But these findings with bees show advanced cognitive capacities are possible with very limited neuron numbers. Scientists are working on relatively simple nervous systems to unravel the mystery of how they create animal intelligence.





Teacher's Notes

Science topics

Calculating speed, animal behaviour, pollination

Maths topics

Calculating percentages, Pythagoras' theorem, coordinates, factorials

Learning outcomes

Age	Students will be able to:
14-16 years old	Calculate percentagesUse Pythagoras' theorem to calculate distancesUse a coordinate system
Duration	Use factorials to calculate permutations
45 minutes	

Keywords

Bee, flight, flower, pollination, foraging, travelling salesman, radar tracking, motion-triggered webcams, permutations, Pythagoras, factorial

Prior learning

What you will need

Students should be able to carry out multiplication and division. They should be familiar with powers and be able to rearrange equations.

- Computer Access with software to produce graphs and tables
- Rulei
- Pencil
- Graph Paper
- Calculator









Introducing the lesson

This lesson is designed to be delivered as either a maths or science lesson. The strategic sampling game is related to the work done at Rothamsted Research on tree diseases.

Provide the students with the recent research information along with the activity sheets. Students can check their results using the answer sheets provided at the end.

In this interactive session the students are going to try to develop a strategy for finding diseased trees within an ash plantation. As with all the other activities, the aim is to make the activity as interactive as possible. So rather than giving the students answers straight away, ask some questions and coach the students in the right direction. The activity is best carried out like the popular game Battleships, with each team ensuring that their board cannot be seen by their opponents.

Explain to the class what a column vector is and how to write one down.

Extension activity

In order to calculate the number of different possible routes that the bees could take to visit the four flowers the scientists used a factorial.

A factorial is represented by the symbol ! and is the product of the multiplication of all the positive numbers equal to or less than it e.g. $3! = 3 \times 2 \times 1$.

Plenary

Discuss with students how they think the bees work out the shortest routes to fly and how they remember the best route. You may want to discuss the importance of bees for our environment and food security, highlighting studies that have shown pesticides may affect the ability of bees to navigate.

Video

Watch the flight of a bumblebee www.youtube.com/watch?v=W2YEzY8tzMU









Curriculum Links

DRAFT Science programme of study for Key Stage 4

Coordination and control in animals and plants

• How different living organisms respond to their environment

Ecosystems

- Components of an ecosystem (abiotic factors and biotic community)
- Relationships among organisms in an ecosystem

DRAFT Maths programme of study for Key Stage 4

Geometry and measures

• Apply Pythagoras' theorem and trigonometric ratios to find angles and lengths in right-angled triangles





Curriculum Links

Further reading and links

Lihoreau, M., Chittka, L. and Raine, N.E. (2010). Travel optimization by foraging bumblebees through readjustments of traplines after discovery of new feeding locations. American Naturalist.

Lihoreau, M et al. 'Radar Tracking and Motion-Sensitive Cameras on Flowers Reveal the Development of Pollinator Multi-Destination Routes over Large Spatial Scales' published in PLOS Biology online on 20 September 2012.

www.plosbiology.org/article/info%3Adoi%2F10.1371%2Fjournal.pbio.1001392

Research groups

Biological and Experimental Psychology Group, Queen Mary University of London

School of Biological Sciences and the Charles Perkins Centre, The University of Sydney **sydney.edu.au/perkins/**

School of Biological Sciences, Royal Holloway www.royalholloway.ac.uk/biologicalsciences/home.aspx

Environment and Sustainability Institute, University of Exeter www.exeter.ac.uk/esi/





Activity 1

Scientists have discovered that bumblebees are able to find the shortest distance between flowers when collecting nectar.

The bees' movements were recorded on a diagram showing the positions and coordinates of the flowers in a flight room.

The flight room was 8.7m wide and 7.3m long.

First the scientists had to find out how long the bees flew for.

They calculated the total time bees spent flying by subtracting the time spent at each flower from the total time taken to complete their foraging.

Flying time = Foraging time - time at flowers

Can you calculate how long the bees flew for?

Question 1: During the first flight (flight A) the bee spent:

- 1. 15 seconds at flower one
- 2. 9 seconds at flower two
- 3. 12 seconds at flower three
- 4. 13 seconds at flower four

The total time the bee spent foraging was 67 seconds.

How long was the bee flying for? Record your answer in seconds.

Seconds

Question 2: During the optimal flight (flight B) the total time the bee spent foraging was 59 seconds.

- 1. Flower three was 11 seconds
- 2. Flower one was 14 seconds
- 3. Flower four was 13 seconds
- 4. Flower two was 10 seconds

How long was the bee flying for? Record your answer in seconds.

Seconds





Activity 2

Can you calculate how far the bees flew?

In order to calculate the distance the bees flew you need to use Cartesian coordinates and Pythagoras' theorem, just like the scientists.

Start by calculating the distance between the nest and the first point the bees discovered.

You will need a right-angled triangle which has the route flown as the longest side of the triangle (the hypotenuse).

Then calculate the length of the two sides that make up a right-angled triangle using the coordinates.

To do this you will need to work out the difference between the coordinates.

For example the coordinates of the nest (3.2;2.3) must be subtracted from the coordinates of the first flower (5.0;6.6).

5.0 - 3.2 = 1.8m

6.6 - 2.3 = 4.3m



Now you can use Pythagoras' theorem: $a^2 + b^2 = h^2$ to work out the distance the bee flew.

 $1.82 + 4.32 = (distance between the nest and first flower)^2 = 4.66m$







Question 1: Below is a diagram of the bee's first flight (flight A).

Calculate the total flying distance of the bee when it was first released from the nest.

After calculating the distance in metres between each flower, round your answer to two decimal places.



Record your final answer in centimetres.

cm



26 of 57



Student

Question 2: Below is a diagram of the bee on an optimal flight (flight B). Calculate the total flying distance of the bee after it had learnt the optimal route to visit the flowers. After calculating the distance in metres between each flower round your answer to two decimal places.



Record your final answer in centimetres.

cm

Question 3: Using the answers from Activity 1.

Calculate how fast the bee flew during each flight. Record your answer in metres per second (m/s or ms⁻¹).

m/s or ms-1







Extension - Activity 3

In order to calculate the number of different possible routes that the bees could take to visit the flowers, the scientists used a factorial.

A factorial is represented by the symbol ! and is the product of the multiplication of all the positive numbers equal to or less than it e.g. $3! = 3 \times 2 \times 1$.

Example: 6! = 6 × 5 × 4 × 3 × 2 × 1 = 720

The factorial of the number of flowers will give the number of different routes there are.

Question 1: How many different routes did the scientists calculate the bees could use to visit all four flowers in the flight room?

Question 2: After the experiments in the flight arena the scientists decided to set up an experiment in a field with five flowers.

How many different routes did the scientists calculate the bees could use to visit all five flowers in the field?





Teacher

Activity 1

Question 1: Flight A - 18 seconds

Question 2: Flight B - 11 seconds

Activity 2

Question 1: Nest to flower 1: $(5.0 - 3.2)^2 + (6.6 - 2.3)^2 = (1.8)^2 + (4.3)^2 = 3.24 + 18.49 = <math>\sqrt{21.73} = 4.66$ m Flower 1 to flower 2: $(7.8 - 5.0)^2 + (6.6 - 0.6)^2 = (2.8)^2 + (6.0)^2 = 7.84 + 36.0 = \sqrt{43.84} = 6.62$ m Flower 2 to flower 3: $(7.8 - 0.4)^2 + (6.6 - 0.6)^2 = (7.4)^2 + (6.0)^2 = 54.76 + 36.0 = \sqrt{90.76} = 9.53$ m Flower 3 to flower 4: $(7.4 - 0.4)^2 + (6.6 - 5.0)^2 = (7.0)^2 + (1.6)^2 = 49.0 + 2.56 = \sqrt{51.56} = 7.18$ m Flower 4 to nest: $(7.4 - 3.2)^2 + (5.0 - 2.3)^2 = (4.2)^2 + (2.7)^2 = 17.64 + 7.29 = \sqrt{24.93} = 4.99$ m Total distance of discovery flight A: 3,298 cm.

Question 2: Nest to flower 3: $(3.2 - 0.4)^2 + (6.6 - 2.3)^2 = (2.8)^2 + (4.3)^2 = 7.84 + 18.49 = \sqrt{26.33} = 5.13 m$ Flower 3 to flower 1: $(5.0 - 0.4)^2 + (6.6 - 6.6)^2 = (4.6)^2 + (0)^2 = 21.16 + 0 = \sqrt{21.16} = 4.60 m$ Flower 1 to flower 4: $(7.4 - 5.0)^2 + (6.6 - 5.0)^2 = (2.4)^2 + (1.6)^2 = 5.76 + 2.56 = \sqrt{8.32} = 2.88 m$ Flower 4 to flower 2: $(7.8 - 7.4)^2 + (5.0 - 0.6)^2 = (0.4)^2 + (4.4)^2 = 0.16 + 19.36 = \sqrt{19.52} = 4.42 m$ Flower 2 to nest: $(7.8 - 3.2)^2 + (2.3 - 0.6)^2 = (4.6)^2 + (1.7)^2 = 21.16 + 2.89 = \sqrt{24.05} = 4.90 m$ Total distance of optimized route B: 2,193 cm.

Question 3: Flight A – 1.83 m/s Flight B – 1.99 m/s

Extension - Activity 3

Question 1: Total possible number of routes with four flowers (factorial 4!) = **24 Question 2:** Total possible number of routes with five flowers (factorial 5!) = **120**





🖬 Glossary

Bumblebee

Any of the 250 species of large, hairy, social bees of the genus Bombus that nest underground.

Factorial

A factorial is represented by the symbol ! and is the product of the multiplication of all the positive numbers equal to or less than it. For example: $5! = 5 \times 4 \times 3 \times 2 \times 1 = 120$.

Foraging

The act of looking or searching for food.

Permutations

A rearrangement of the elements of a set.

Pollinate

To transfer pollen from an anther to the stigma of (a flower).

Pythagoras' theorem

The theorem applies to any right-angled triangle and states that the square of the length of the hypotenuse (the side opposite the right angle) is equal to the sum of the squares of the other two sides: $h^2 = a^2 + b^2$.

Travelling salesman problem

This is a mathematical problem that poses the following challenge: Given a list of cities and the distances between each pair of cities, what is the shortest possible route that visits each city exactly once and returns to the city of origin?





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Recent Research

Teacher

Mark-recapture – predicting and protecting our pollinators using variance and standard deviation

Biologists often need to estimate the number of animals in a particular place, perhaps to see if the numbers are decreasing because of a disease, to decide whether conservation methods are working or to find out whether one habitat can support more animals than another.

In doing this, the biologists face the problem of how to count or estimate the number of animals in a certain area if they can't all be seen.

A common method used to overcome this is called mark-recapture.

A few examples of the application of the mark-recapture method by Rothamsted scientists are:

- Estimating bumblebee population sizes. Accurate estimations of population sizes are required in order to keep track of the decline of these essential plant pollinators.
- Estimating carnivorous beetle populations. Certain carnivorous beetles are known to be effective at controlling insect pests that damage crops. The size of the beetle population can provide us with an estimate of the likely success in reducing an insect pest and thus the probable amount of crops that will be damaged by them.





Scientists at Rothamsted Research track bumblebee population size, monitor their survival, record changes in habitats and investigate the impact of disease and pesticides.

One of the projects they are carrying out is trying to find the link between the abundance of flowers in farmland and the survival of bumblebee nests. Bumblebees are essential for pollination of flowers and crops, and like all bees, bumblebees eat only pollen and nectar from flowers. We know quite a lot about their tastes, and different bumblebee species like different flowers and they need a continual succession of flowers within range of their nests throughout the spring, summer and early autumn.

Apart from food, bumblebees also require sites where they can nest. In order to understand the bees' survival, the scientists must work out the number of bumblebee nests which fail due to predators, parasites and disease; the distribution and seasonal changes in flowers in the landscape; and the nutrients, such as carbohydrate and protein, required for successful development of a bumblebee nest. They must also establish the numbers of nests and how they develop in different landscapes.

This will enable the scientists to develop a model to predict how much pollination bumblebees carry out and to find out what effect growing more flowers can have.

Further reading



An integrated model for predicting bumblebee population success and pollination How do you track a honey bee? www.bbc.co.uk/news/magazine-23448846









Science topics

Populations, sampling

Maths topics

Rearranging equations, statistics, variance, standard deviation

Learning outcomes

Age	Students will be able to:
14-18 years old	 Estimate population size using the mark-recapture principle Produce statistics including calculation of variance and standard deviation
Duration	
45 minutes	

Keywords

Mark-recapture, population size, bees, habitats, pollination, pesticides, conservation, estimation, distribution, variance, variability, standard deviation

Prior learning

What you will need

- Stack of bee cards (each set of bee cards must contain more than 10 cards and to work well should contain at least 20 bee cards)
- Small stickers to mark the cards
- Calculator
- Paper or a flipchart (to keep track of results and to work out means and variances)
- Question and answer sheets



Students should be able to carry out multiplication, division and rearrange equations.





Introducing the lesson

This lesson is designed to be delivered as either a maths or science lesson. The lesson is suitable for students taking GCSE statistics or as an extension lesson for maths or science.

Provide the students with the recent research information along with the activity sheets. Students can check their results using the answer sheets provided at the end.

The lesson first of all introduces both the concept of the mark-recapture technique used to estimate population size and the equation. Students are required to rearrange the equation to work out the number of bee cards they have been provided with.

The next stage of the lesson involves the students repeating this activity and calculating the variance and standard deviation of their results.

If the students are not sure about how to calculate the variance and standard deviation, first have a discussion about what they think they represent, and then show them how to calculate it.

Plenary

Discuss the following questions with the students.

Question 1 – Why do we say that we are estimating the number of bees in the field by mark-recapture methods? Why is the result uncertain, and not exact?

Question 2 - What assumptions do we make when estimating the number of bees this way?

Question 3 – What would happen to our estimate if bees discover that the traps are warm, safe, foodfilled places to spend the night and they become 'trap-happy'?

Question 4 – What would happen to our estimate if bees hate the traps, and avoid them after first being caught?

Question 5 – What would happen to our estimate if some bees are caught by predators or die between the first and second round of trapping?

Plenary question answers

Answer 1. We assume that the proportion of bees in the second round of trapping is equal to the proportion of bees in the field that are marked. However, this is unlikely to be exactly true. Statistically (subject to the assumptions below) the expected value of the marked proportion in the second round is the true proportion in the whole population (we say the estimate of the proportion is unbiased), but any one sample is likely to differ from this. We call this difference the sample error.

Answer 2. One important assumption is that, in both rounds of trapping, all bees in the field are equally likely to be caught. Only if this is true are we justified in our assumption that the proportion of bees caught in the second round that are marked is an unbiased estimate of the true proportion of marked bees in the whole population. In the exercise with cards, this assumption is met only if the cards are thoroughly shuffled.





Answer 3. This would be a violation of our assumption that all bees are always equally likely to be caught (previously caught bees are more likely to be caught again). We shall therefore overestimate the proportion of bees in the field that are marked, and so underestimate the total number.

Answer 4. This is also a violation of the assumption set out in (2) above. This time we shall tend to underestimate the marked proportion, and so overestimate the total number.

Answer 5. This highlights the importance of our assumption that we know how many bees are actually marked. The estimated proportion of bees that are marked is still an unbiased estimate of the true proportion. The total number of marked bees in the field will be smaller than we think (unless, for some reason, marked bees are never caught by predators), because some have been eaten by predators, and we will overestimate the population.





DRAFT Science programme of study for Key Stage 4

Ecosystems

- Levels of organisation: species, population, community, ecosystem, biome and biosphere
- Components of an ecosystem (abiotic factors and biotic community)
- Relationships among organisms in an ecosystem

Human interactions with ecosystems

- The importance of biodiversity in ecosystems
- Identifying and classifying local species and using keys
- Measuring the distribution, frequency and abundance of species in a range of habitats and explaining outcomes in terms of abiotic and biotic factors
- Measuring changes in the distribution and abundance of organisms as a way of measuring and monitoring change in ecosystems
- Positive and negative human interactions with ecosystems
- The biological challenges of increasing food yield using fewer resources

DRAFT Maths programme of study for Key Stage 4

Number

- Apply systematic listing strategies, including use of the product rule for counting
- Estimate powers and roots of any given positive number
- Calculate with roots, and with integer and fractional indices

Algebra

• Simplify and manipulate algebraic expressions (including those involving surds and algebraic fractions)

Statistics

- Infer properties of populations or distributions from a sample, while knowing the limitations of sampling
- Apply statistics to describe a population





Teacher

Further reading and links



For more information on the application of mark-recapture methods www.pitt.edu/~yuc2/cr/main.htm

Research groups

Sustainable pollination services for UK crops – Dr Koos Biesmeijer, University of Leeds https://secure.fera.defra.gov.uk/beebase/downloadDocument.cfm?id=360

Modelling systems for managing bee disease: the epidemiology of European foulbrood – Dr Giles Budge, Food and Environment Research Agency

www.nerc.ac.uk/research/funded/programmes/pollinators/pollinators-budge.pdf

Investigating the impact of habitat structure on queen and worker bumblebees in the field – Dr Claire Carvell, NERC Centre for Ecology and Hydrology

www.nerc.ac.uk/research/funded/programmes/pollinators/pollinators-carvell.pdf

An investigation into the synergistic impact of sublethal exposure to industrial chemicals on the learning capacity and performance of bees – Dr Chris Connolly, University of Dundee **www.nerc.ac.uk/research/funded/programmes/pollinators/pollinators-connolly.pdf**

Linking agriculture and land use change to pollinator populations – Professor Bill Kunin, University of Leeds

www.nerc.ac.uk/research/funded/programmes/pollinators/pollinators-kunin.pdf

Urban pollinators: their ecology and conservation – Professor Jane Memmott, University of Bristol www.nerc.ac.uk/research/funded/programmes/pollinators/pollinators-memmott.pdf









Research groups

Impact and mitigation of emergent diseases on major UK insect pollinators - Dr Robert Paxton, Queen's University of Belfast

www.nerc.ac.uk/research/funded/programmes/pollinators/pollinators-paxton.pdf

Unravelling the impact of the mite Varroa destructor on the interaction between the honeybee and its viruses – Dr Eugene Ryabov, The University of Warwick **www.nerc.ac.uk/research/funded/programmes/pollinators/pollinators-ryabov.pdf**





Student







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Student









Biologists often need to estimate how many animals are in a particular place. This could be to see whether the numbers are declining because of a disease, to decide whether conservation methods are working or to find out whether one habitat can support more animals than another.

This is not always easy. Some large animals can be counted from aircraft. But how would you count the number of bees in a field?

A common method is called **mark-recapture.** Bees are caught live in traps, marked (e.g. with enamel paint), and then released. Then there is a second round of trapping. We assume that the proportion of the second group of trapped bees that are marked is equal to the proportion of marked bees in the whole population.

To get an idea how this technique works we can play a card game.

	Rules
What you will need	 You are not allowed to count the number of cards in the pack (that's just cheating and would not be possible in real life).
 A pack of cards (each with a picture of a bee) Stickers 	2. Take the top 10 cards (these are the bees that you have caught), and mark each bee (add a sticker).
	3. Shuffle the pack of cards thoroughly.
	 Take the top 10 cards again (these are the bees caught in the second round of trapping).
	 Count how many of the second set of caught bees were already marked with a sticker.
	If we think about this technique mathematically we can write an equation to work out how many bees there are.
	• The number of bees marked = <i>n</i> (in this case 10)
	 The number of marked bees caught in the second round of trapping = m (marked bees plus unmarked bees)
	 Total number of marked bees in the field (we know this, because we marked them in the first round of trapping) = M

• The total number of cards (bees) = N

$\frac{m}{n} = \frac{M}{N}$

Can you rearrange this equation to estimate N from the numbers that you know?

Turn over for the answer.







Activity 1

Answer

$$N = \frac{nM}{m}$$

From the numbers you know, estimate the total number of bees (cards in the pack).

Estimated total number of bees (N) =

You are still not allowed to count the number of cards!

Variance and standard deviation

To get a better estimate you will need to repeat the mark-recapture technique a number of times and use some statistical techniques. You will learn how to work out the variance and standard deviation of your results.

Variance and the closely related standard deviation are measures of how spread out a set of results is. This is known as distribution (it is not how spread out the bees are in the environment). In other words, variance and standard deviation are measures of variability.

The variance (σ^2) is calculated as the average squared deviation of each number from its mean. For example, for the numbers 1, 2, and 3, the mean is 2 and the variance is:

$$\sigma^2 = \frac{(1-2)^2 + (2-2)^2 + (3-2)^2}{3} = 0.667$$

The formula (in summation notation) for the variance in a population is:

$$\sigma^2 = \frac{\sum (X - \mu)^2}{N}$$

- \sum (summation) is the sum of all values in the range of the series
- X represents each value in the range
- μ is the mean
- **N** is the number of scores

The standard deviation is the square root of the variance.





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Variance and standard deviation continued

What are the advantages of variance and standard deviation? What effect does squaring have?

Squaring each difference makes them all positive numbers (to avoid negatives reducing the variance).

And it also makes the bigger differences stand out. For example $100^2 = 10,000$ which is a lot bigger than $50^2 = 2,500$.

But squaring them makes the final answer really big, and so un-squaring the variance (by taking the square root) makes the standard deviation a much more useful number.

Activity 2: Repeating the mark-capture card game

- 1. Play the game as you did in Activity 1.
- 2. Repeat the game five times.
- 3. Record the estimated total number of bees (N) each time you repeat the game.
- **4.** After five games calculate the mean estimate, the variance and the standard deviation of your estimate.

Mean estimate = _____

Variance =

Standard deviation =

Activity 3: Changing the sample size

- Change the number of cards that you mark and repeat the mark-recapture game five times.
- How does the sample size affect your estimates?

Now you can count the number of cards.

Total number of bees =

How close was your initial estimate to the actual number of bees?







🖬 Glossary

Bumblebee

Any of the 250 species of large, hairy, social bees of the genus Bombus that nest underground.

Conservation

The protection, preservation, management, or restoration of wildlife and of natural resources.

Habitat

The area or environment where an organism or ecological community normally lives or occurs.

Foraging

The act of looking or searching for food.

Mark-recapture

Organisms are caught live in traps, marked (e.g. with enamel paint), and then released. Following a second round of trapping it is assumed that the proportion of the second group of trapped organisms that are marked is equal to the proportion of marked organisms in the whole population.

Pesticide

A chemical used to kill pests, especially insects.

Pollinate

To transfer pollen from an anther to the stigma of (a flower).

Population

All the organisms that constitute a specific group or occur in a specified habitat.

Standard deviation

A measure of dispersion in a frequency distribution, equal to the square root of the mean of the squares of the deviations from the arithmetic mean of the distribution.

Variance

A measure of distribution that is calculated as the square of the standard deviation.





Recent Research

pest that could threaten UK farming.

and biodiversity.

Experimental design - find out what Spanish slugs prefer to eat using factorials and statistics

Spanish slugs

© John Innes Centre

When Dr Ian Bedford, an expert in insects, spiders and other creepycrawlies, noticed an unusually high number of slugs in his garden he

A foreign cannibalistic slug has been identified in the UK for the first time. Known as the Spanish slug, this species is an aggressive crop

Based on the damage the Spanish slug has caused in other countries, scientists are concerned about the potential effects on food crops

Researchers from the John Innes Centre and UK universities are investigating the problem and are calling on the public to help – through the citizen science project SlugWatch – find out how far and

wide the Spanish slug has spread around the UK.

didn't realise he had stumbled on a foreign invasion. The large brown slugs seemed to have appeared

from nowhere and were eating almost everything in their path. Dr Bedford, head of the John Innes

Centre's Entomology Facility, was intrigued by the high number and aggressiveness of the slugs and decided to investigate.

He found massive numbers of the brown slug in his garden, with at least 50 to 100 slugs on the lawn.

He also noticed it wasn't just plants that were being attacked by this species of slug, they were actually eating things like dead animals, other slugs, snails, and even shells.

Fortunately, in his role at the John Innes Centre, an international centre of excellence in plant science and microbiology, Dr Bedford had an eye out for potential crop pests in the UK. He resolved to find out what this slug was, suspecting it could be a foreign species called the Spanish slug (Arion vulgaris) which has become a problem in northern mainland Europe in recent years, and has been nicknamed the 'killer slug' because of its cannibalistic tendencies.

Dr Bedford contacted a slug expert, Dr Les Noble from Aberdeen University, who was interested in what was happening with slugs in the UK. Due to unusually wet weather, there had been a population boom in a species called Arion flagellus, which is commonly known as the Spanish Stealth Slug, leading to large numbers of the slugs being reported.

But both the numbers and behaviour seen by Dr Bedford were slightly different to what was recorded for Arion flagellus and were more similar to what had happened in Europe, where a species commonly known as just the Spanish slug, Arion vulgaris, had moved into Scandinavia and had absolutely decimated huge numbers of crops. There were even instances of cars having crashed because slugs had been eating roadkill and had made the roads slippery.







Recent Research

Dr Bedford sent some specimens to Aberdeen for Dr Noble to examine. They were identified and confirmed, for the first time in the UK, as Arion vulgaris.

The presence of this aggressive species is bad news. The biggest threat to crops is in the spring when the buried eggs hatch. With few predators, a voracious appetite and the ability to lay around 400 eggs each, the slugs multiply quickly, can cause huge damage to crops and can push out other slugs and snails, thus dominating an area. The crops that are threatened by this slug include oilseed rape and potatoes.

Scientists also want to look at the environmental impact because we know that where this slug appeared in northern Europe, other native species of slugs disappeared. The scientists want to find out what is actually happening – for instance, do these invasive species bring in parasites that kill off similar slug species but don't actually kill off their original host?

When foreign slugs invade they interbreed with native species, which could lead to new hybrid species in the UK. It might be possible in future to use genetic sequencing to see whether this is happening.

Research links



The cannibalistic foreign slug which could threaten UK crops and biodiversity [Reference/ webpage no longer available – January 2017]







Science topics

Experimental design, choice tests, animal behaviour, crop pests

Maths topics

Factorials, algebra, statistics

Learning outcomes

Age	Students will be able to:
14-18 years old	 Understand the concepts of replication, randomisation and blocking in experimental design
	 Design an experiment that maintains neighbour balance of treatments
Duration	 Propose a mathematical rule that calculates the number of repeats required in an experiment with neighbour balance for any number of treatments
45 minutes	

Keywords

Experimental design, replication, randomisation, blocking, neighbour balance, factorial, slug, lettuce, choice test

Prior learning

What you will need

Students should be able to divide and multiply factorials and rearrange equations.

- A board with marked-out slug arenas
- Plastic numbered discs representing different lettuce varieties









Introducing the lesson

This lesson is designed to be delivered as either a maths or science lesson.

In this session the students are going to design some 'statistically sound' experiments. The activity should be as interactive as possible with plenty of opportunity for students to explore the possible solutions. It will be far easier for the students to see what is happening if they use the board representing the experimental dish.

Explain what experimental design is and why it is essential. The students will then try and design a specific experiment, the slugs puzzle. First explain the biological problem that you want them to design the experiment for. When they have solved the puzzle, let the students have a go at questions 1–3.

If students have not encountered factorials before, choose a suitable point in the middle of the lesson to go over the principle of a factorial.

Plenary

Discuss with students other ways that the experiment could be designed and the difficulties in creating a perfect experiment. Ask students to explain the equations used to calculate the permutations of lettuce variety arrangements.

Homework

Take a look at the information sheet on experimental design and prepare a 15-second summary to explain to the class what experimental design is and why it is essential.









DRAFT Science programme of study for Key Stage 4

Coordination and control in animals and plants

• How different living organisms respond to their environment

Ecosystems

- Levels of organisation: species, population, community, ecosystem, biome and biosphere
- Components of an ecosystem (abiotic factors and biotic community)
- Relationships among organisms in an ecosystem

Human interactions with ecosystems

- The importance of biodiversity in ecosystems
- Identifying and classifying local species and using keys
- Positive and negative human interactions with ecosystems
- The biological challenges of increasing food yield using fewer resources

Evolution, inheritance and variation

• The diversity of organisms and how each adapts to a particular environment as a result of evolution

DRAFT Maths programme of study for Key Stage 4

Number

- Apply systematic listing strategies, including use of the product rule for counting
- Estimate powers and roots of any given positive number
- Calculate with roots, and with integer and fractional indices

Algebra

• Simplify and manipulate algebraic expressions (including those involving surds and algebraic fractions)

Probability

- Apply the property that the probabilities of an exhaustive set of mutually exclusive events sum to one
- Understand that empirical unbiased samples tend towards theoretical probability distributions, with increasing sample size
- Calculate the probability of independent and dependent combined events, including using tree diagrams and other representations, and know the underlying assumptions
- Calculate and interpret conditional probabilities through representation using expected frequencies with two-way tables, tree diagrams and Venn diagrams









Further reading and links

Experimental Design at Rothamsted Research sheet









Experimental dish



Lettuce varieties









Student

The slugs and lettuces problem

One of the distinctive behaviours of the Spanish slug is its wide-ranging diet. In order to discover just what crops are under threat from the Spanish slug, scientists will need to carefully design an experiment.

Experimental design originated in the Statistics department at Rothamsted Experimental Station when Ronald Fisher joined in 1919. Rothamsted has a history of research on slugs and, using their approach to experimental design, we can plan an investigation to find out just what the slugs will eat.



Slug and Lettuce © Rothamsted Researc

Experimental design

First we need to learn a little bit about experimental design. Ronald Fisher was employed to analyse data from crops grown in fields with different manures. Most field trials up to this time were laid out in strips, with a single treatment of each type of manure applied to each strip. He realised that there were problems with this design, for instance, what if one strip got more water than the others?

Ronald came up with three principles: replication, randomisation and blocking.

Replication – repeating treatments to allow estimation of variability and hence improve precision of experiments

Randomisation – assigning treatments to plots at random (with equal probability) for unbiased estimates of treatment effects

Blocking - to account for known features or trends in experimental material

You will be designing experiments to answer scientific questions.

The scientific question

Which of seven lettuce varieties is most resistant to a particular species of slug?







of lettuce

The Experiment (A Choice Test)

- 1. Seven leaf discs, one of each variety, will be placed around a circular dish (= block)
- 2. Several such dishes (= replicates) will be set up simultaneously
- 3. A single slug will be added to each dish and left to feed for a fixed period of time
- 4. The amount of each leaf disc remaining will then be measured
- 5. The variety with the most leaf area remaining overall will be declared the most resistant to slug damage

Your task is to decide the order of the varieties around each of the dishes.

An example (randomised) order for one dish is shown in Figure 1. Each dish acts as a block as each variety is tested under the conditions in that dish and in relation to a single slug. However, in your experiment we will not simply produce a different random order for each dish but will try to achieve what we will call neighbour balance for the varieties. In this way the experiment can test each lettuce variety with different neighbours, in case they put the slug off eating that variety.

Neighbour balance. An experiment is neighbour balanced if each treatment (variety, in this case) has every other treatment as its neighbour on an adjacent plot an equal number of times.

For example in Figure 1 treatment 5 is next to 7 and 2. It also needs to be next to 1, 6, 3 and 4. You could achieve this by moving treatment 5 to between 1 and 6 in one dish and between 3 and 4 in another dish. However, the other treatments would not be next to each of the other. How will you solve this?

Now have a go at designing an experiment yourself!







Student



The Experiment (A Choice Test)

Question 1. What is the minimum number of dishes required so that each variety occurs next to each of the others once overall? Can you see a pattern for making your choice?

Why would this neighbour balanced arrangement offer an advantage over simple randomisation within dishes?

Question 2. Do you think this number of dishes will be enough to answer the scientific question adequately? (Think about what might happen to the slugs!)

What might you do to increase the number of dishes but still maintain the neighbour balance?

How would you then arrange the varieties in the extra dishes?

Question 3. What would the minimum number of dishes be if you only wanted to compare six varieties?

Can you write down a mathematical rule (or rules) for determining the minimum number of dishes required for any number of varieties?

Remember each variety must be next to the other varieties an equal number of times.

Hint: You will need to use factorials in your equation. A factorial is represented by the symbol ! and is the product of the multiplication of all the positive numbers equal to or less than it e.g. $3! = 3 \times 2 \times 1$.



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Teacher

The slugs and lettuces problem





Question 1. Using Figure 1 as a starting point, you should find that three dishes are sufficient to ensure that each variety appears next to every other variety once overall.

Explanation: For example, variety 3 occurs next to 4 and 6 in the first dish, next to 1 and 7 in the second, and next to 2 and 5 in the third. The arrangement in the second dish was obtained from that in the first dish by taking every other variety, starting with variety 5 and moving clockwise (5, 1, 3 ...) and placing them in the new order. The varieties were taken off until there were none left on the dish and placed in the order they were removed. The arrangement in the third dish was obtained similarly but by taking every third variety (5, 6, 7 ...).

This arrangement ensures that there is no bias in the experiment with respect to the varieties. If, for example, variety 1 was particularly tasty to the slugs, and this variety was always next to varieties 2 and 6 (as in the first dish), the amount eaten of these two varieties might be unduly affected, leading to incorrect conclusions about these varieties compared to variety 1. In fact, we would probably also rotate the dishes so that variety 5 is not always in the same position within the dishes.







The slugs and lettuces problem

Question 2. Generally, we would use more than three dishes.

Explanation: With only three dishes, if one slug were to die, for example, we would not have much information left with which to answer the question. Biological material is also inherently variable (different leaves of the same variety may be slightly more or less attractive due to differing levels of chemical compounds; different slugs will have different feeding rates or levels of hunger, etc.). Higher replication (numbers of dishes/slugs in this case) will give a more accurate and precise estimate of the average amount of each variety remaining. A more reliable conclusion as to which variety is most resistant will therefore be reached.

You would therefore want to repeat the three dishes at least once (i.e. use six dishes) to maintain the neighbour balance. However, we would not repeat the same set exactly. Instead, we would simply randomise the varieties in the positions in the fourth dish and then choose the fifth and sixth arrangements according to the same pattern as for the first three. So we might start, for example, with 2764135 and then obtain 2615743 and 2456371. Again, we would probably rotate the dishes so that varieties (e.g. especially variety 2 here) do not always occur in the same positions.

The exact number of replicates to use depends on how much variability is expected and how big a difference between the varieties it is important to detect. In practice, statisticians use mathematical formulae to estimate the appropriate number of replicates for an experiment.

Question 3. You should have found that with six varieties you would need five dishes to achieve neighbour balance. However, when there is an even number of varieties, we can only achieve neighbour balance by having each variety next to each other variety twice overall.

Explanation: The same pattern of selection used for odd numbers doesn't work. It works for odd numbers because the requirement is that every variety must be surrounded by two other varieties – so when you take one variety from 3, or 5, or 7, or 9, or 11, and so on, you then have an even number of varieties left to choose from. But, for an even number of varieties, taking one leaves you with an odd number. So after choosing pairs there will always be one variety left over to pair with one already chosen.

We therefore need a different rule for odd and even numbers of varieties.

To work it out we will use an equation with factorials.

Let **V** represent the number of varieties to be tested. The number of ways of choosing two varieties, from **V** varieties, we'll call: ${}^{v}C_{2}$. The way this is written is shorthand for writing permutations (how many different ways we can arrange things) of a combination.

It looks complicated, but don't worry we'll go through it step by step.

The small V shows how many varieties we are working with and the 2 represents the neighbour pairs e.g. with six varieties the number of ways of choosing two varieties to be neighbour pairs = ${}^{6}C_{2}$.





The slugs and lettuces problem

The number of ways of choosing two varieties is actually a longer equation featuring factorials: where

${}^{v}C_{2} = \frac{V!}{(V-2)! \times 2!}$ and $V! = V \times (V-1) \times (V-2) \times ... \times 1.$

The top row of the equation, v!, gives the number of different ways you could arrange the varieties. But you do not need to use every different arrangement. The bottom row of the equation takes into account the fact that one arrangement has two pairs for each variety e.g. in Figure 1, variety 5 is paired with varieties 7 and 2.

This equation can be simplified to:

$$C_2 = \frac{V \times (V - 1)}{2!}$$

To calculate the number of dishes required, we will call this number N, we need to rearrange the equation. For odd numbers of varieties:

$N = C_2 / V$

e.g. V = 7 requires N =
$${}^{7}C_{2} / 7 = {}^{v}C_{2} = \frac{7!}{5! \times 2!} / 7 = \frac{7 \times 6}{2} / 7 = 3$$

But for even numbers of varieties this rule doesn't work because it doesn't give us a whole number of dishes.

e.g. V = 6 would give N =
$${}^{6}C_{2} / 6 = \frac{6!}{4! \times 2!} / 6 = \frac{6 \times 5}{2} / 6 = 2.5$$

In fact this rule gives us half the number of dishes required, so for even numbers of varieties the required rule is:

$N = 2 \times {}^{v}C_{2} / V$

e.g. V = 6 requires N = 2 ×
$${}^{6}C_{2} / 6 = 2 \times (\frac{6!}{4! \times 2!} / 6 = 2 \times (\frac{6 \times 5}{2} / 6) = 5$$

The multiplication by two here for even numbers of varieties results from the need to see each variety next to every other twice rather than once overall.

In attempting this task, as well as solving a mathematical puzzle, you have encountered the fundamental statistical experimental design principles of replication, randomisation and blocking.





🖬 Glossary

Blocking

To account for known features or trends in experimental material.

Entomology

The scientific study of insects (slugs are not insects).

Factorial

A factorial is represented by the symbol ! and is the product of the multiplication of all the positive numbers equal to or less than it. For example: $5! = 5 \times 4 \times 3 \times 2 \times 1 = 120$.

Neighbour balance

An experiment is neighbour balanced if each treatment has every other treatment as its neighbour on an adjacent plot an equal number of times.

Permutations

How many different ways we can arrange things.

Randomisation

Assigning treatments to plots at random (with equal probability) for unbiased estimates of treatment effects.

Replication

Repeating treatments to allow estimation of variability and hence improve precision of experiments.

Spanish slug

Arion vulgaris, a new species of slug in the UK that varies in colour between bright orange and reddish brown and can grow to a size between 8cm and 15cm when they have reached maturity.





